PERIODICA POLYTECHNICA SER. ARCHITECTURE VOL. 35, NOS. 1–2, PP. 79–89 (1991)

CENTRAL PROJECTION OF HELIX

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Received: April 20, 1991.

Abstract

Acquiring descriptive geometry skills through the curricula of architectural studies can play an important role in preparing students for efficient use of 3 dimensional computer programs. Though nowadays most of the geometric constructions are built-in procedures mainly in 2D CAD programs, users must be able to navigate with 3D models, getting the right projections of them. Architects and construction engineers, whose basic professional communications are drawings, need to use the new medium adroitly. With computers they have to make 'theoretical' decisions rather than to construct exact projections of objects in a traditional way. This article presents a problem in descriptive geometry in order to show the changing role of descriptive geometry teaching for architectural students.

Construction — Drawing — Communication

Architectural drawings bring the ideas of designers close to clients and other specialists involved in a project. These drawings must be clear and adequate enough to describe space and objects existing first in the mind of the designers. In this process there are standardized and free elements of representation. Freehand drawing provides a more artistic, subjective description of a building or space than a technical one. The great influence of technical drawing on art began in the Renaissance when Filippo Brunelleschi, the Florentine architect, developed the mathematical law of perspective. His contemporaries based their spatial concepts on a scientific foundation.

Appearance of perspective on paintings had a main role in providing good optical effects in the perception of space. Later when architects documented their works for clients and the public more widely than before, perspective construction influenced architectural drawings, too.

Even though perspective pictures are more recognizable than orthographic ones, orthographic drawings kept their importance in three dimensional description because they could be constructed much easier. Orthographic projection is an efficient method of transferring three-dimensional

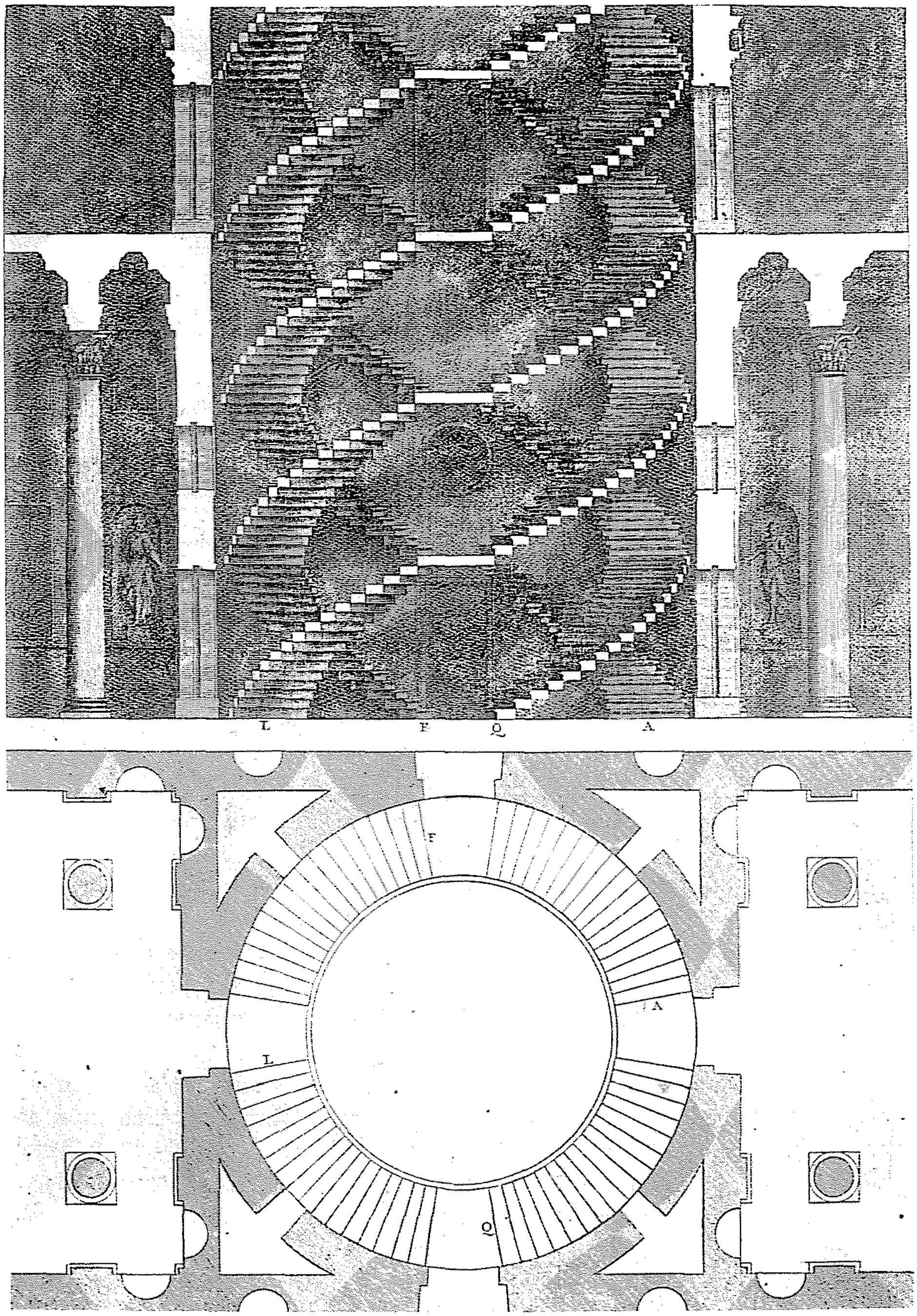

Fig. 1.

objects into two dimensions by parallel (and perpendicular) projection. These drawings are intended to transfer ideas to others and sometimes to check solutions of design problems graphically. Graphic representation has always been an essential part of the process of design. Descriptive geometry gives the rules of projections and helps to develop spatial thinking. Nowadays steps of accurate construction can be completed on computer controlled devices (screens, plotters and printers). These new ways of communication through graphic computer programs expect users to be able to have a fast overview of computer created drawings. Instead of drawing tiresome figures users must be able to choose the most informative figures. Once a model is entered, figures can be created rapidly with computers, unfortunately bad figures also. Mistakes also can be produced and reproduced faster than with a traditional way of drawing. To avoid this danger users must have good skills to check a drawing's reliability rather than to be able to construct nice, accurate curves of shapes.

Pictures of Spiral Curves as Building Elements

Spiral staircases are not only decorative but impressive part of buildings. In building construction guides one can find plans and sections of the structure of spiral staircases. (*Figs. 1, 2*) LEONI G., (1715)

These drawings are supposed to give details of constructions rather than just show an outline of shapes.

Hans Vredeman de Vries, a Dutch painter, shows a spiral staircase in an open space (1604). (*Fig. 3*). This example with its construction lines shows the construction of complexity of revolving and lifting faces of a staircase.

Geometrical Definition and Orthographic Projection

For practical reasons one should learn the construction of a helix first as a geometrical curve. Helical motion is the result of a winding and a progressing motion. Points performing helical motion describe helical path or helix. Angles of tangents to this 3 dimensional curve are constant. Since the most descriptive and easily constructable/understandable drawings are pictorial views of spatial elements, they are preferred. Pictorial views can be constructed by parallel projection. Orthogonal (axonometric or multiview) projection is a comfortable way to show a helix. Drawings will have different features according to the direction of projection (*Fig. 4*).

For the example of parallel projection let us use orthographic projection, which means that the projectors are perpendicular to the picture

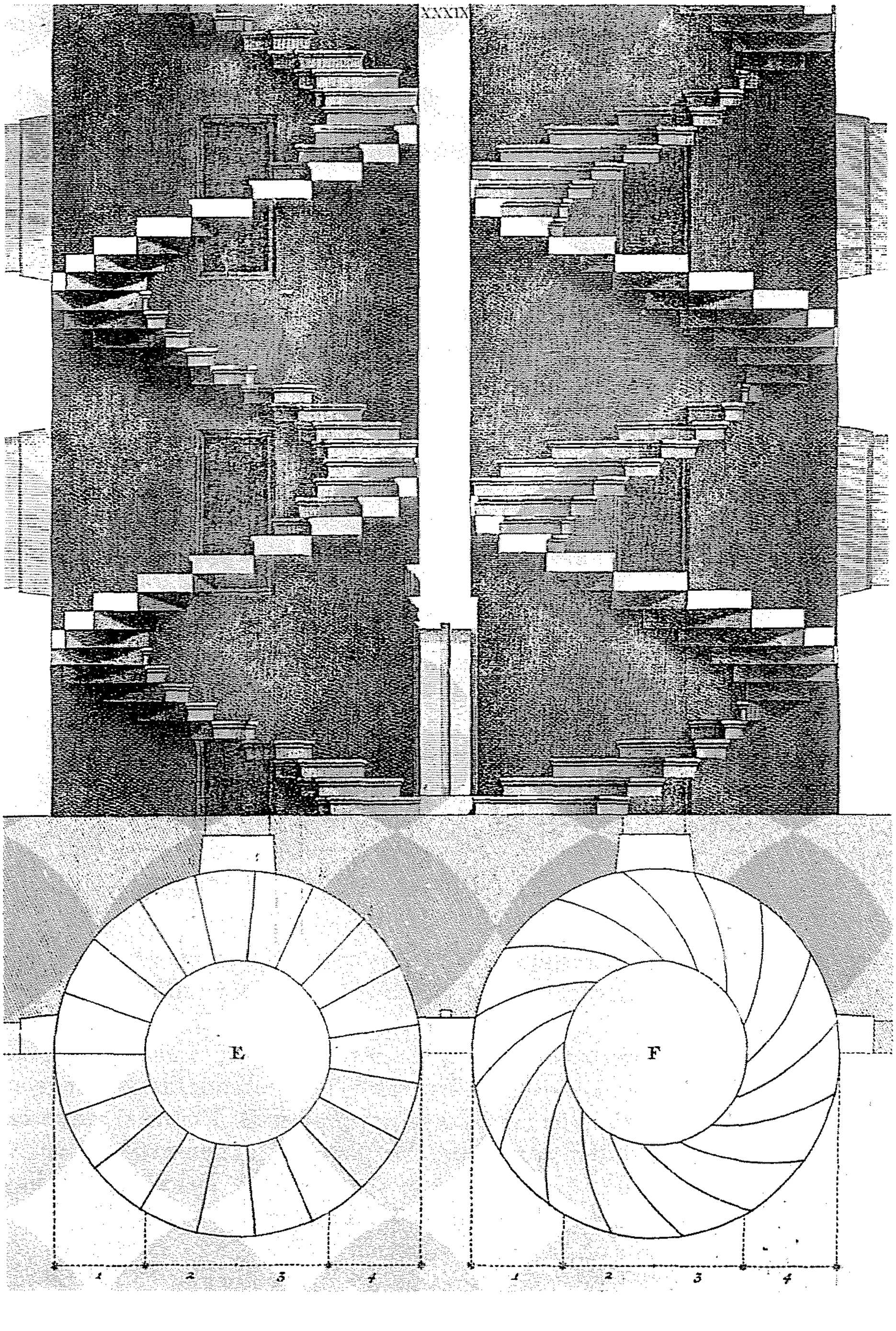

Fig. 2.

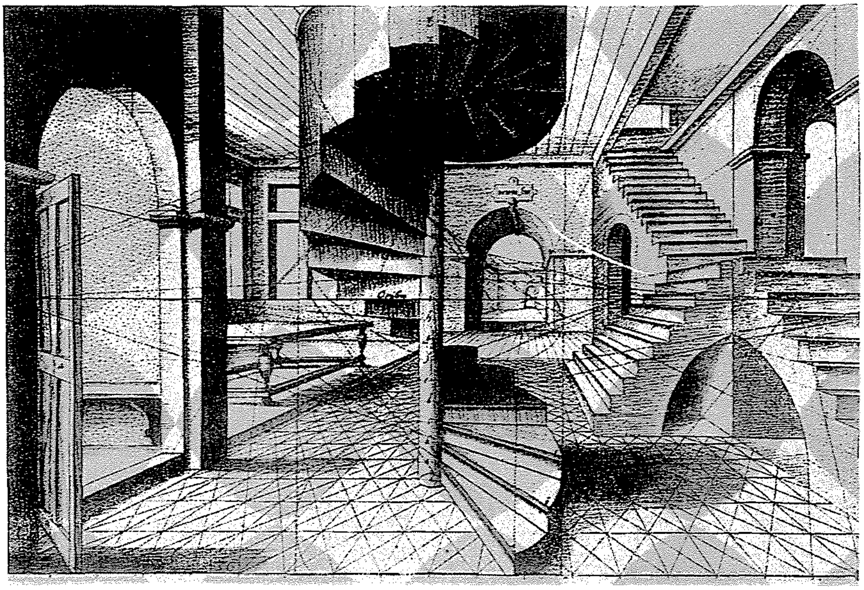

Fig. 3.

plane. In the first case (*Fig. 4a*) the axis of the helix intersects the picture plane at an angle. At a certain point changing this angle with the same increment the picture will be a sine curve. This happens when the axis of the helix is parallel to the picture plane. This situation is between the state of F and G. An interesting and characteristic orthographic view of this helix occurs when the curves become peaky. (*Fig. 4d*) This happens when the angle between the axis of helix and the picture plane equals the angle of pitch of the helix. Then the projectors of the peaky points are tangents to the curve. Obviously, all the contour points on the left side of the curve are peaky since we used parallel projection.

Central Projection

Sometimes it is difficult to appreciate points, details of design or the actual appearance of a building or objects from orthographic projections such as a plan or an elevation or even an axonometric view. In architectural practice the preferred perspective presents as nearly as possible the actual appearance. Thus this technique is very important in the work of artists, architects, engineers, interior designers and other specialists, making it possible to view the project as a finished object before it is manufactured. Com-

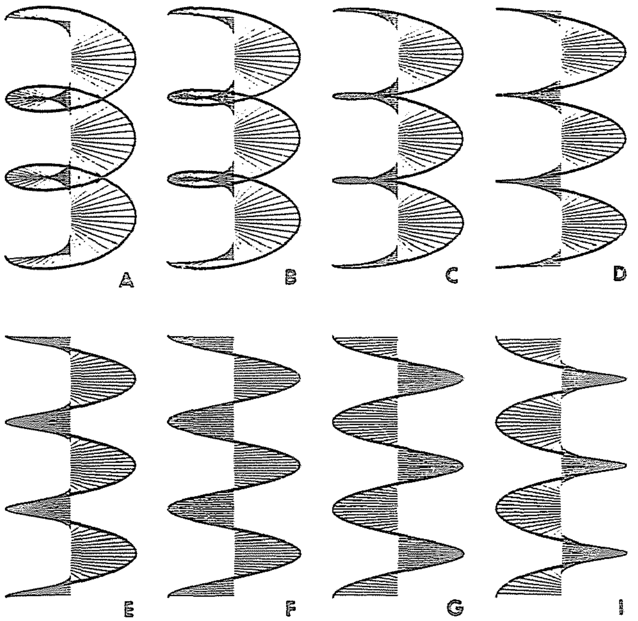

Fig. 4.

puters gained ground in this field by helping to make perspective drawings much easier than before.

Since the rule for creating perspective drawings is obvious, the question is where to set the centre of projection in order to get the same peaky contours on a picture of a helix as we got in an axonometric view. Do we have those kinds of points in perspective also, and if so under what conditions? (LŐRINCZ, P. – PETRICH, G. (1976)).

Examining *Fig. 5*, a computer generated view of projection, we find that the above-mentioned points appear on different sides of the contour-line.

We can get those peaky points on the drawing at those points of the helix where the projector is tangential to the curve of the helix. Consider a helix with its axis q, radius of its cylinder is r (*Fig. 6*). A centrum of projection is C, lines AC and BC are tangential to the helix at point A and B, respectively. (A top view of this case is shown in *Fig. 7*).

Let us take a point called D halfway between A and B on the curve of the helix. Points A_x, B_x, C_x and D_x are projections of A, B, C and D, respectively, on the base plane. There is a base circle with the radius of r in this plane which is the base of the cylinder having the helix on its

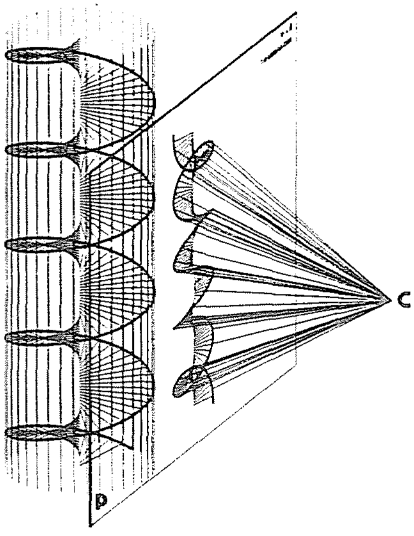

Fig. 5.

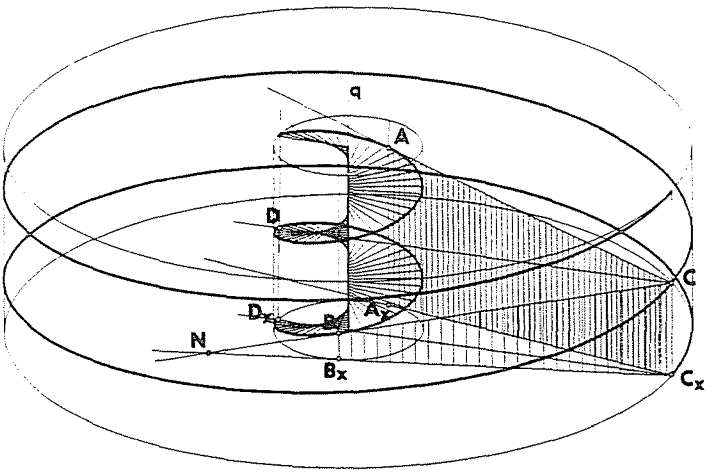

Fig. 6.

surface. Let us denote by α the angle of $QC_xA_x = QC_xB_x$ (*Fig 7*). Since $k = C_xA_x = C_xB_x$ is tangential to the base circle, we can write an equation in the form

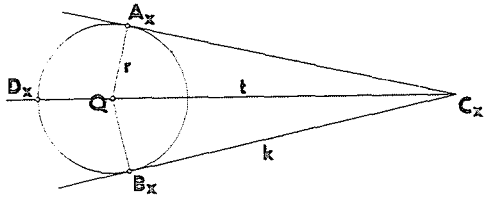

Fig. 7.

$$k = r/\operatorname{tg}(\alpha). \tag{1}$$

Moreover, the length of DA on the helix is equal to the length of AC, and similarly BD is equal to BC. (*Fig. 6*) This statement is evident since the AA_xD_xD surface of the cylinder is developable into a tangential surface of the cylinder as AA_xC_xC. Similarly BB_xD_xD surface of the cylinder is developable into BB_xC_xC. In both cases points D and D_x get in C and C_x, respectively. This means that the length of $k = A_xC_x$ is equal to the circumference of the circular arc of A_xD_x and the length of $k = B_xC_x$ is equal to the circumference of the circular arc of D_xA_x. The angle belonging to the circular arc of A_xD_x is equal to $\pi + (\pi/2 - \alpha)$ where α is the angle of $QC_xA_x = QC_xB_x$ as mentioned above. An equation for k can be written in the form

$$k = r^*(\pi + \pi/2 - \alpha) \tag{2}$$

Since tangential points can be on the next pitches of the helix also, a more general form of the equation (2) is:

$$k = r^*(n^*\pi + \pi/2 - \alpha). \tag{3}$$

where n is any non-negative integer.

In virtue of equations (1), (2)

$$r/\operatorname{tg}(\alpha) = r^*(n^*\pi + \pi/2 - \alpha). \tag{4}$$

Dividing both sides with r and bringing α on the left side:

$$\operatorname{cotg}(\alpha) + \alpha = n^*\pi + \pi/2. \tag{5}$$

Solving the equation (5) for α we have the following:

n	a (in radians)	a (in degrees)	k	t
1	0.219	12.547	4.711 $*r$	4.603 $*r$
2	0.129	7.376	7.854 $*r$	7.790 $*r$
3	0.091	5.240	10.995 $*r$	10.950 $*r$
4	0.071	4.066	14.137 $*r$	14.102 $*r$
5	0.058	3.323	17.279 $*r$	17.250 $*r$
6	0.049	2.810	20.420 $*r$	20.396 $*r$
...	...	...	...	

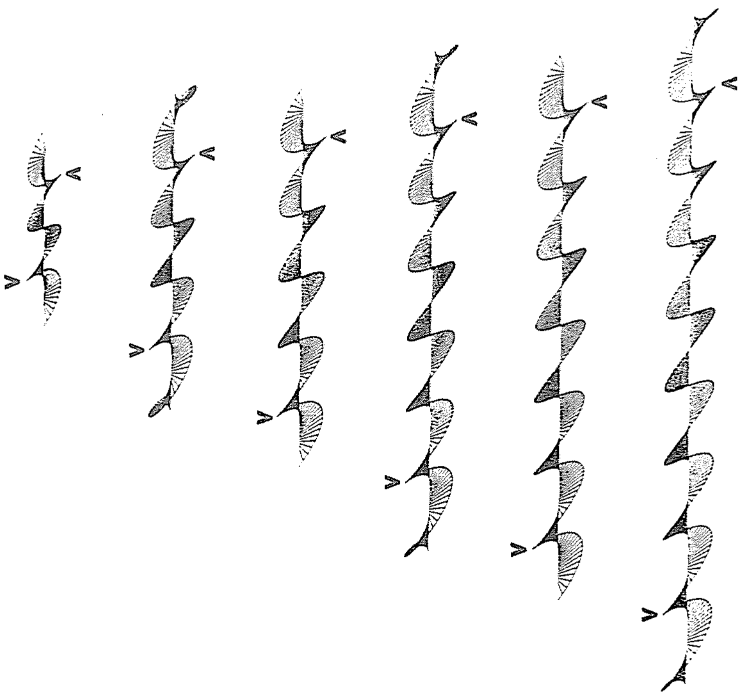

Fig. 8.

Let us turn triangle ACB so that AC and BC remain tangential to the helix. In this case point C will trace a helix also. This helix is the locus of those points from which the original helix can be projected with

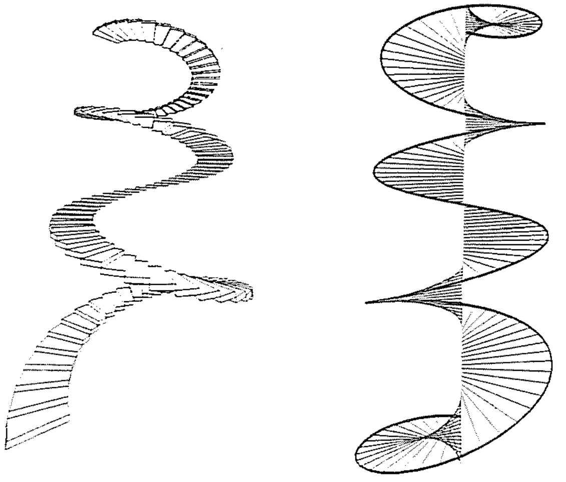

Fig. 9.

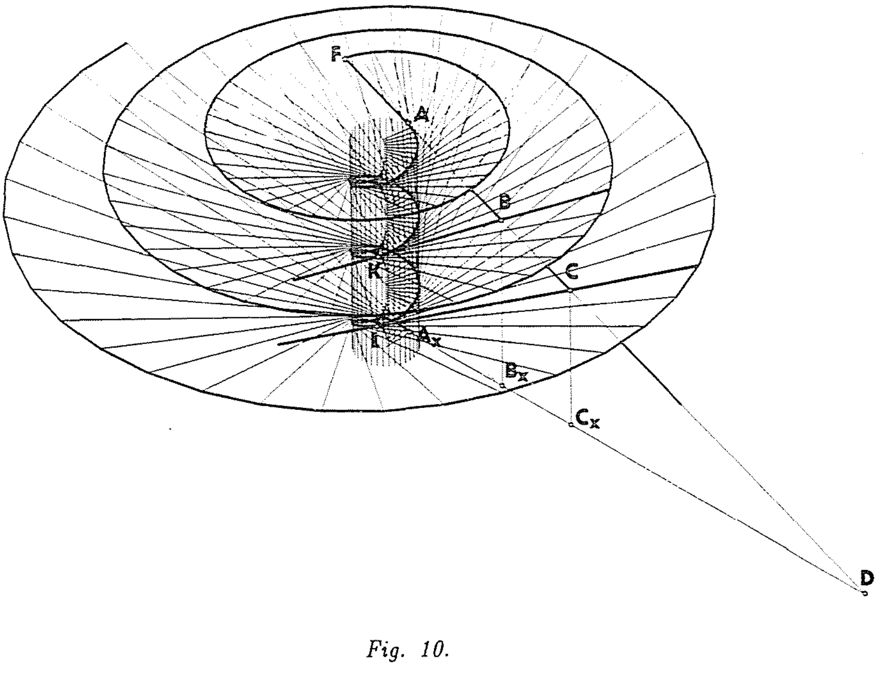

Fig. 10.

peaky points on it. There are projections of the same helix from different viewpoints (as given in the table) on *Fig. 8.* Those peaky points move away from each other while the centre of the projection moves away from the axes of helix by the value of t. *Fig. 9* shows an example of central projection when picture plane is not parallel to the axis of helix. Compare this figure with *Fig. 5*.

An approach to finding centers for projection from which the curve of a helix will be drawn with peaky points might be formulated in another way, too. Draw a tangent to the helix to point A on the helix. Denote FD the tangential line of the helix at point A (*Fig. 10*). Then draw all the tangents to the curve of each point. This range of lines will provide an developable tangential surface to helix. Construct the intersection of this surface and the tangential line FD. Piercing points B and C only are shown on the figure, but with the same procedure one can get further points. These piercing points (B, C, etc.) will be the required points for the center of projection. Points K, L, etc. are the points whose projections become a peak on the curve besides A's projection.

Conclusion

This looking round on a central projection of a geometrical curve as an architectural element may require a new way of studying curves other than how to construct them. Computers can construct drawings fast and accurate enough for us; what we really have to do is to choose the right parameters of objects and viewpoints.

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